

# Spinors and Gravity: A Reinterpretation and Extension of the Dirac Equation

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## Abstract

A new interpretation of the Dirac equation is introduced where the Dirac matrices become dynamic fields and the keeper of geometric information. This is facilitated through the use of a bilinear, rather than a quadratic lagrangian and an auxiliary field that is a kind of dual that mirrors the evolution of the electron wavefunction. These new gravitational fields appear through a “spin 1” type of lagrangian reminiscent of electromagnetism and generate a kind of “square root” of general relativity. This lagrangian gives a greatly enlarged gauge group so it become essential to define carefully the gauge invariant part, or “reality,” of the fields. In packet form, this reality of the electron and electromagnetic fields are shown to follow geodesic motion for a metric defined by a simple quadratic function of the these gravitational Dirac matrix fields. This geometric interpretation is made possible by a Higgs-like coupling term that distinguishes gravity from electromagnetism at low energies creating the nonlinear features familiar from general relativity while allowing evolution on a flat background and giving global conservation laws.

## 1 Introduction

The Dirac equation is the fundamental description for fermions in quantum theory. It is typically derived in terms of causality arguments and the need for an equation of motion that is first order in time, as was Dirac’s approach, or, more formally, in terms of representation theory of the Lorentz group. These arguments are discussed many places [1] [2] [3]. While this is a powerful description and has led to the first inclination of the existence of antiparticles, it has its own problems. Negative energy solutions have had to be reconciled by Dirac’s original hole theory or through the second quantization operator formalism. Most are so steeped in this long established perspective and impressed by its successes that it gets little discussion.

A monumental problem today is that of “unification” of quantum theory and gravity. There are formal perturbative approaches to this and some string theory approaches as well. In quantum field theory we often start with a single particle picture as a “classical field theory” and then use canonical quantization or path integral methods. For this reason, it is good to have a thorough understanding of the classical theory to be built upon. We will show that, by making some rather formal changes in traditional lagrangians, new results can appear.

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The main purpose of this article is to illustrate an alternate interpretation of the Dirac equation. In the course of it, we will make gravity look much more like the other bosonic fields of nature and give a true global conservation law (that is generally elusive in GR). Our motivation begins with a reconsideration of the spinor transformation laws and the role of representation theory. This approach will greatly expand the gauge invariance of the system. In place of the metric  $g_{\mu\nu}$  as the keeper of gravitational information, we will let the  $\gamma$  matrices become dynamic fields and evolve. Our motivation for this is that, for vector fields, the metric explicitly appears in each term and variation of it, gives the stress-energy tensor. The only object directly coupling to the free Dirac fields is  $\gamma$ . Additionally,  $\gamma^\mu$  bears a superficial resemblance to  $A^\mu$  and the other vector bosons. Since  $g \sim \gamma\gamma$  we might anticipate that the spin of this particle is one rather than two as is for the graviton theories which are based explicitly on  $g^{\mu\nu}$ . A unification of gravity in some analogous fashion to electroweak theory would benefit from having its field be of the same type.

There has been work from the geometric algebra perspective before [5] in trying to reinterpret the Dirac and Pauli matrices as physically meaningful objects. Since the author has labored in isolation for many years searching for a physical meaning for the apparent geometric nature of physical quantities this did not come to his attention until recently. However, there are significant differences in the approach presented here. Most importantly, one has a new notion of gauge freedom as it relates to the reality expressed by particle fields (i.e. the full gauge independent information associated with it). Coupling destroys the ability to associate the full “reality” of the electron with the wavefunction. We will see that this can get much more entangled when one includes gravity and, with the exception of phase information, the only consistent notion of a particle’s reality comes from the locally conserved currents that can be associated with it but here will involve multiple field functions.

In this article we only discuss these as classical theories in a 4D spacetime. Of course, the motivation is for this to lead to a general quantum theory. There is a lot of work on reinterpretation of quantum theory as a deterministic one. Everyone who works on this has his favorite approach. The author here is no exception and have in mind resolution that is consistent with the theory in [9] that gives QM in a suitable limit. The motivations behind the following constructions is not just to get some insight on unification but to take steps to resolve some of the fundamental contradictions of quantum field theory, such as Haag’s theorem, and to give a solid justification for the calculations of field theory that have been successful.

## 2 Transformation Rules

The theory of spinors arose naturally out of Dirac’s algebraic attempts to reconcile causality with the first order equations that seem to describe nonrelativistic electrons. Interestingly, Schrödinger originally attempted the, later named, Klein-Gordon equation to describe electrons but could not get the fine structure right [3]. He settled on a diffusion-like equation that was first order in time and second order in spatial derivatives. Pauli adapted it to include spin but, as for most such equations, signal propagation speeds diverge. Dirac introduced a pair of spinors and a linear first order operator that when “squared” gave the Klein-Gordon equation for each component, thus ensuring causality.

His treatment introduces a set of  $\gamma_{ab}^\mu$  matrices that are considered fixed and constitute representations of the  $SL(2, \mathbb{C})$  group which is a two-fold covering group of the  $SO^+(3, 1)$  group. More explicitly, this gives a map of complex valued bi-spinors  $\begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix}$  to real 4-vectors so that each 4x4 complex matrix action corresponds to a Lorentz transformation and compositions among these is

preserved by this mapping. In the humblest of terms, we can decompose a general free state  $\psi_a$  into a basis of free progressive wave solutions  $e^{ik_\mu x^\mu} u_a(k)$  where we can define a general Lorentz transformation  $\Lambda_\nu^{\mu'}$  through the coordinate *and* algebraic action  $S(\Lambda)_{ab}\psi_b(\Lambda x)$ . We define this action so that the current  $j^\mu$  is transformed by a boost and interpret it as the actively boosted free plane wave of positive energy. Note that  $S(\Lambda)_{ab}\psi_b(x) \neq \psi_a(\Lambda x)$ . Such simple coordinate boost of the argument gives a boosted current but is generally interpreted to mix positive and negative energy states in an unphysical way. We will see that in our later approach this is not necessarily so.

The Dirac Lagrangian has a (seemingly) symmetric form

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \quad (1)$$

where  $\bar{\psi} = \psi^\dagger\gamma^0$ . This inconvenient  $\gamma^0$  is generally considered necessary to give Lorentz invariance. We can see that without it we would get inconsistent equations of motion for  $\psi$  and  $\psi^*$  if we vary them independently.

The operator  $S(\Lambda)_{ab}$  performs a transformation of  $\psi_a$  so that the Lagrangian is invariant and the resulting current is boosted as  $j'^\alpha(x') = (\psi'(x')^\dagger\gamma^\alpha\psi'(x')) = ((S\psi(x))^\dagger\gamma^\alpha S\psi(x)) = (\psi(x)^\dagger S^\dagger\gamma^\alpha S\psi(x)) = (\psi(x)^\dagger\gamma'^\alpha\psi(x)) = \Lambda_\beta^\alpha(\psi^\dagger(x)\gamma^\beta\psi(x)) = \Lambda_\beta^\alpha j^\beta(x)$ . The Dirac theory allows us to think of the complex 4-spinors  $\psi_a$  at each point as indicating the local direction of the local current of the particle corresponding to it. To achieve this it has been necessary to introduce negative energy solutions.

The negative energy solutions are reinterpreted as positrons and given a positive mass through the details of canonical quantization since they are generally deemed undesirable. Other conservation laws such as the conservation of probability (which arise from the same global phase symmetry that give mass and charge conservation) have similar problems. In an “emergent” theory of quantum mechanics we do not need a probability operator (or any operators at all). The probabilities arise from measurements with the kinds of macroscopic yet still quantum matter that constitutes the classical world [9]. In this approach, the initial data and evolution equations generate their dynamics in a deterministic fashion and the probabilistic features arise from the long lived partitioning of the classical world into subsets indexed by the delocalized objects that interact with it. Details of when this is a consistent procedure are discussed in ref.[9]. For this reason, we do not seek to validate or build upon arguments that start with an “interpretation” of particular expressions since we ultimately expect the evolution and interactions to independently determine the expressions that give all observable results.

One of the frustrating aspects of the Dirac equation as it stands is that it is not clear how we should alter its form in general coordinates. One can use the local frame approach and assume the Dirac matrices are members of the same representation in each one. A spinorial connection then indicates how nearby spinors are related as a consequence of geometry. If we allow the matrices to become functions of space and time with only the spacetime indices changing this gives a simple approach but then it is not clear how we recover local KG evolution of each component and what the locally boosted fields should be. If we continue with the spinor approach and let the  $\gamma^\mu(x)$  matrices be fixed and alter the spinor fields instead then we need a transformation that is a kind of “square root” of the Lorentz vector transformation. This is how we get the actively boosted solutions in flat space. In curved spacetime, there is no global notion of a boost so the former perspective seems more valuable. Ultimately, we specify a configuration by the spacetime metric and the fields on it but the metric will be a function of the  $\gamma^\mu$  matrix fields (and some associated dual fields) that only give geodesic motion below some energy bound.

In the early days of the Dirac equation, interpretations have evolved from a proposed theory of electrons and protons to that of electrons and positrons with positrons as “holes” in an infinitely full electron “sea” to that of electrons with positrons as electrons moving “backwards in time.” The first interpretation failed because the masses of the positive and negative energy parts are forced to be equal. The second was introduced out of fear that the negative energy solutions of the Dirac equations would allow a particle to fall to endlessly lower energies. The last was introduced as a computational tool. The negative mass solutions were to be reinterpreted as positive mass with negative charge. Necessary computational fixes associated with this idea are subtly introduced through the anticommutation relations used in the field theory approach to fermions and the properties of the supposed ground state [4]. If we are going to seek a classical field theory approach to this problem we need another mechanism.

If we restrict to positive energy states for the noninteracting case, then no negative states ever exist. In the case of interactions, we will only generate negative energy states if the interactions allow it and, even then, there is no reason to believe that a finite system with conserved energy would have some particle fall to arbitrarily low energies. We will see that our theory can accommodate such negative energy states without catastrophe. These other states will form a kind of indistinguishable mirror world of dynamics precluding any bias to have solutions fall to arbitrarily low energy. Since the only place that the size of a particle’s charge enters the lagrangian is through the coupling term, positrons of positive mass can be handled by introducing a separate wavefunction and coupling to the electromagnetic field with an opposite signed charge. This subtracts from the historical meaning of the negative solution as pointing to antiparticles but is a simple and functional solution. Here we assume the  $\gamma$  matrices are those of the Dirac representation. Standard treatments allow any selection of 4x4 matrices that represent the  $SO^+(3,1)$  group. Here we choose a specific representation because we are going to let the  $\gamma$ ’s be fields and let these other choices be a kind of gauge freedom until some interaction restricts us to a specific subset.

The Dirac Lagrangian has a (seemingly) symmetric form

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \quad (2)$$

where  $\bar{\psi} = \psi^\dagger\gamma^0$ . This is generally considered necessary to give Lorentz invariance. The Dirac matrices satisfy the condition

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu} \quad (3)$$

where  $\eta = \text{Diag}(-, +, +, +)$ . This suggests that we could view the metrical properties of the space as encoded in  $\gamma$  rather than invoking a metric  $\eta$ . The metric has ten independent parameters at each point and  $\gamma$  has  $4 \cdot 10$  or  $4^3$  parameters, depending on chosen symmetry constraints but we need to satisfy  $4^4$  equations. If we trace the suppressed spin indices then there are only 10 equations and a general metric can be encoded in the  $\gamma^\mu$  set. However, eqn. 3 is the identity we require to convert the Dirac equation into a KG one that demonstrates causality in each component. This is a loose end in deriving geodesic motion for a packet to show that we get observed motion in the classical GR limit and an important consideration in what follows.

In anticipation of a future unification theory one cannot help but notice the greater similarity of  $\gamma_{ab}^\mu(x)$  to  $A^\mu(x)$  and the other vector boson fields than any of these to the metric  $g_{\mu\nu}$ . For now we simply leave this as constant but accept that it can have its own transformation properties as a one-vector. In contrast, all the “spinor” labels are considered as having only scalar transformation properties. The bispinors  $\psi_a$  now transform as scalars. To emphasize their new properties and that

they still have a collective reality as a four-tuple of functions we term it a “spinplet.” The mixed objects  $\gamma_{ab}^\mu$  we consider a vector object with extra labels and, by analogy, label it a “vectorplet.”

There are some surprising implications of this. The equations are unchanged but the transformation properties are now different. Since the  $\gamma_{ab}^\mu$ ’s can vary with position, we expect a much larger equivalence class of electron-gravity field pairs,  $\{\psi, \gamma\}$ , that correspond to the same underlying reality. We can boost the system by  $\Lambda_\alpha^\mu \gamma^\alpha$ . This gives the same  $\psi_a$  fields at every point but the physically measurable  $j^\nu$  currents are altered. Of course we still have the traditionally boosted solutions  $S(\Lambda)\psi^{(0)}(\Lambda x)$  that have this same current so we have a degeneracy in the pairs  $(\Lambda\gamma, \psi^{(0)})$  and  $e^{i\phi}(\gamma, S(\Lambda)\psi^{(0)})$  and all other states with the same current and net phase. This is not the result of a discrepancy in the active vs. passive coordinate transformations we observe in a fixed representation but an additional degeneracy in the equivalent physical descriptions. We have only used the current  $j^\mu$  to distinguish states and we expect that there will be some other conserved quantities that will physically subdivide this set into distinct equivalence classes. Since there are so many degrees of freedom in the set of  $\gamma_{ab}^\mu(x)$ ’s we anticipate that the set is still significantly enlarged.

### 3 Reality and Gauge

The AB effect gives a simple example of how the “reality” of an electron is not sufficiently described by the wavefunction of the electron itself. In this case, the current is a function of both  $\psi$  and  $A$ . This suggests we might want a more formal distinction of particle reality versus the types of fields in a lagrangian. Traditionally, the discussion of gauge freedom supplants such a discussion. It is clear that gauge equivalence is just another name for being in an equivalence class. We are going to be interested in pushing the notions of equivalence further so a clear distinction will be helpful.

In flat space without gravity or interactions, we can consider packets of field that are widely separated based on type. These can then evolve separately and the type of field and the reality implied by it are synonymous. There can still be some gauge freedom but the packets and any interesting properties that one might observe are contained in the same support. The observables are, at best, the gauge invariant properties such as stress-energy or current. Allowing interactions, this reality gets complicated in two ways. Firstly, the conserved currents may now involve aspects of more than one kind of field and second, there are now constraints that must be obeyed. These are generally defined by elliptic PDEs such as  $\nabla \cdot E = \rho$  that are propagated by the dynamic equations.<sup>1</sup>

If we now include gravity in the form of a  $\gamma_\mu$  field that has some gauge freedom that mixes with the reality of the wavefunction  $\psi$  then we cannot make the above separation. The gravitational field is everywhere so no isolation of packets is possible. The reality of the electron is now a function of  $\psi$  and any  $\gamma$ -like fields that have global extent. This is in contrast with the case where the gravitational information is completely specified in the  $g^{\mu\nu}$  field. Since this has no gauge freedom beyond that of coordinate changes, the packet motion of a wavefunction is affected by it yet the reality of the electron is still entirely determined by the values of  $\psi$  in the packet itself.

For the case where multiple fields determine a single reality, when is it really viable to call one set of quantities the “electron current” versus some combination of quantities that strictly depend

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<sup>1</sup>This is purely a classical theory of delocalized fields so we do not have the problem of “self-energy” or the “particle not feeling its own fields.” In the many body case, the fields presumably are made of many constituent ones with only the “center of mass” motion as visible. This allows us to have a wavefunction of charged particle that does not spread under the influence of the field generated by it, as in the classical particle case [?].

on multiple types of fields? In the case of the Dirac and electromagnetic field, the density of the field is only a function of  $\psi$  so that we have at least one component of the 4-current that is entirely specified by the wavefunction. This allows us to uniquely associate  $j^\mu$  with the electron field  $\psi$  and so call it the “electron-current.” The stress-energy terms similarly have  $T^{tt}$  as a simple function of  $\psi$  alone. If every conserved quantity can be associated this way, we have a well-defined mapping between the fields and conserved quantities. If we are interested in more exotic lagrangians than can be formed by the “minimal” prescriptions from the free quadratic cases, we will need to be mindful of the possibility that the currents may not necessarily be so associated with one particular field. In such a case the phrasing “energy density of the electron” may be meaningless unless one wishes to define the electron to be labelled by the particular energy density and then relabel the wavefunction  $\psi$  as some abstract descriptor with some other name that plays a role in defining the reality of the electron.

Although this discussion may feel somewhat pedantic, it is important to make this distinction and not get trapped in the vague lore that sometimes accompanies discussions in physics. For example, it is often said that we must have “manifestly invariant” lagrangians to get relativistically consistent results. This is not true not only in the obvious sense that they can be rearranged in a nonobvious invariant form. One can conceivably write down a set of fields that gives a class of solutions whereby the degrees of freedom and invariance is with respect to the observers built of other physical fields. Here we can imagine inducing a set of “physical coordinates” based on local packets of long lasting separated objects that define a grid. With the right time evolution parameterization, we would expect the form of the equations to be invariant with respect to such a coordinate set. The overall class of equivalent solutions should allow for local field changes that induce independent observable current changes with the appropriate degrees of freedom for the observed dynamic freedom of the system. In general, we only need observers to see the world with such symmetry (such as Lorentz) but it need not hold with respect to the coordinates. As long as the constituent fields of the observers and the external reality “covary” together, then the observers see exactly the same thing. Allowing such dynamics enlarges the equivalence classes at the cost of a more complicated relationship between coordinates and observable reality.

## 4 Lagrangians

Generally we seek a quadratic free field lagrangian and then gauge and Lorentz invariant couplings between them. The Dirac lagrangian is usually presented in the superficially symmetric form

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \quad (4)$$

where  $\bar{\psi} = \psi^\dagger\gamma^0$ . The appearance of the  $\gamma^0$  is displeasing if we are to interpret the  $\mu$  indices as spacetime indices. This particular form is often considered important because it gives a positive definite probability density. In an “emergent” approach to quantum theory where the probabilities are defined by the evolution equations in a deterministic fashion, this is not important. Probability will automatically be conserved by the normalization over the resultant paths that bifurcate the observed paths of recording devices as indexed by the delocalized coordinates [9] regardless of whether there is a “nice” operator that describes it. More importantly, we need the eom of  $\psi$  and  $\psi^*$  to be consistent. This dictates that the  $\gamma^0$  appear in this expression. By using a representation where  $\gamma^0\gamma^\mu\gamma^0 = \gamma^\mu$  the variations of the action give equivalent equations of motion.

To achieve a lagrangian that is manifestly invariant using this “vector-plet” interpretation we introduce an auxiliary field  $\phi$  that, in flat space, can be chosen to be  $\psi^\dagger\gamma^0$ . For the usual Dirac

equation this condition is propagated. One should wonder if this will give a true isomorphism with physical results. We are interested in the propagation of conserved quantities as mass, charge... and some local phase information. This brings us to a subtle point. Even in nonrelativistic quantum mechanics, the “reality” of interacting particles is not completely given by the corresponding fields themselves. This is most clearly observed in the AB effect. Often this is viewed as an important example of topology and gauge in physics. It is more simply understood as an expression of the electron current being not simply a function of the electron wavefunction alone. A similar property is observed in the London skin depth in superconductors. The only way an electron current can obtain rotational flow is through the vector field  $\vec{A}$  or through the appearance of discrete vortices. The moral here is that angular momentum, among other conserved quantities, are defined by a collective set of fields so it makes no sense to associate with one particular particle. “Spin” is now a kind of angular momentum that exists through the collective local reality of this new vector-plet graviton and two fermion spinplet fields. By abandoning this usual concept of a spinor we will obtain an isomorphic theory that has significant generalizations.

## 4.1 Bilinear Modification

To resolve the complications arising from the hidden  $\gamma^0$  in the usual Dirac lagrangian, let us replace  $\bar{\psi}$  with an associated yet independent field  $\phi$  and see when it evolves in a consistent fashion when we simplify to the Dirac representation. Consider the Dirac-limiting lagrangian density we can choose using only the complex valued  $\psi$ ,  $\phi$  and  $\gamma^\alpha$  (with  $g^{\mu\nu}$  an implicit function of it) is of the form

$$\mathcal{L} = i(\phi_a \gamma_{ab}^\mu \partial_\mu \psi_b - \partial_\mu \phi_a \gamma_{ab}^\mu \psi_b) - 2m\phi_a \psi_a \quad (5)$$

For constant  $\gamma$ 's chosen to be the Dirac representation, then variation  $\delta\phi$  yields  $i\gamma^\mu \partial_\mu \psi - m\psi = 0$ . Variation by  $\delta\psi$  yields  $-i(\partial\phi)\gamma^\mu - m\phi = 0$ . If we choose  $\phi_a = \gamma_{ab}^0 \psi_b^*$  then this is equivalent to the Dirac equation solution for  $\phi$ .

When we consider the gauge equivalent states this introduces some additional complications. For example, if the support of  $\psi$  and  $\phi$  are disjoint then there is no net mass or current density. Such a state is evidently a vacuum despite the nontrivial values of the functions and evolution equations. Here we see that our notions of the physical meaning we attach to functions as describing the reality of a particle is less trivial than usual.

So far we have not explicitly included any measure or metric and the action of  $\nabla_\mu \gamma^\nu$  is ambiguous without it. We can make formal definitions of these by using eqn. 3 as a guide. The functions  $g^{\mu\nu} = \det(-4^{-1} \text{Tr}_{ac} \gamma_{ab}^{(\mu} \gamma_{bc}^{\nu)})^{-1}$  and  $g_{\mu\nu} = \text{Inv}(-4^{-1} \text{Tr}_{ac} \gamma_{ab}^{(\mu} \gamma_{bc}^{\nu)})$  to define the metric in terms of  $\gamma$  are evidently complicated when explicitly constructed but they do give us trial definitions for  $g^{\mu\nu}(\gamma)$  and its inverse in terms of  $\gamma^\mu$  that can specify a completely general metric field. Another possible objections is that the form of  $\gamma^\mu$  with indices raised as a contravariant object is opposite that of the covariant form that  $A_\mu$  enters the lagrangian especially the interaction terms  $q\bar{\psi}\gamma^\mu A_\mu\psi$  which gives us pause when considering the possibility of treating  $\gamma^\mu$  and  $A_\mu$  as analogous fields where no a priori metric exists.

Since we are interested in a theory that includes electrons, positrons, photons and gravity with the electromagnetic and gravitational fields on an equivalent footing we will need to make a further modification. It will be convenient to let the natural form of  $\gamma$  be lowered index object  $\gamma_\mu$  and introduce a contravariant sister field  $\lambda^\nu$  that generates  $g^{\mu\nu}$  in the same fashion that  $\gamma_\mu$  generates  $g_{\mu\nu}$ . It is not automatic that these be inverse functions despite the suggestive notation

but we will show that they do so in sufficiently low energy cases. We expect the following relations to be able hold in the flat space limit

$$g^{\mu\nu}\delta_{ac} = -2^{-1}\{\lambda^\mu, \lambda^\nu\} = -\lambda^{(\mu}, \lambda^{\nu)} \quad (6)$$

$$g_{\mu\nu}\delta_{ac} = -2^{-1}\{\gamma_\mu, \gamma_\nu\} = -\gamma_{(\mu}, \gamma_{\nu)} \quad (7)$$

It is very important to distinguish between this case, which arises in deriving the Klein-Gordon results that demonstrate causality for the Dirac components and the traced result. The arbitrary metric field  $g_{\mu\nu}(x) = -8^{-1}\text{Tr}\{\gamma_\mu(x), \gamma_\nu(x)\}$  can be defined in terms of  $\gamma_{ab}^\mu(x)$ 's but the untraced result for  $g_{\mu\nu}(x)\delta_{ac}$  cannot. This will be central to what follows.

We like to have the metric appear explicitly in all the terms of the lagrangian for the reason it gives us something to vary in obtaining a conservation law for stress-energy. One way to do this is

$$\mathcal{L}_e = i(g^{\mu\nu}\phi_a\gamma_{\mu:ab}\partial_\nu\psi_b - g^{\mu\nu}(\partial_\mu\phi_a)\gamma_{\nu:ab}\psi_b) - 2m\phi_a\psi_a \quad (8)$$

where the colon separates spacetime from scalar indices. We define  $g^{\mu\nu} = -4^{-1}\text{Tr}\lambda^{(\mu}, \lambda^{\nu)}$ . The evolution equations are given by the variations  $\delta\phi$

$$i(g^{\mu\nu}\gamma_{\mu:ab}\partial_\nu\psi_b + g^{\mu\nu}\nabla_\mu(\gamma_{\nu:ab}\psi_b)) - 2m\psi_a = 0 \quad (9)$$

$$ig^{\mu\nu}\gamma_{\mu:ab}\partial_\nu\psi_b + 2^{-1}ig^{\mu\nu}(\nabla_\mu\gamma_{\nu:ab})\psi_b - m\psi_a = 0 \quad (10)$$

and  $\delta\psi$

$$ig^{\mu\nu}(\nabla_\mu\phi_b)\gamma_{\nu:ba} + 2^{-1}ig^{\mu\nu}\phi_b(\nabla_\mu\gamma_{\nu:ba}) + m\phi_a = 0 \quad (11)$$

so that  $\phi$  evolves as  $\psi$  with  $m \rightarrow -m$  and  $\gamma \rightarrow \gamma^T$ .<sup>2</sup>

Since we are about to determine the motion of the conserved gauge invariant stress energy associated with the fields and it is deeply connected with geometry, we make a brief segue to derive this conserved quantity. A general action contains both a lagrangian and a measure that can be related to the metric

$$S = \int d^4x \mathcal{L} \sqrt{-g}. \quad (12)$$

Incorporating general relativity, the lagrangian density is generally written

$$\mathcal{L} = \frac{1}{2\kappa}R(g) + \mathcal{L}_{\text{fields}} \quad (13)$$

where  $\kappa = 8\pi G$  and the first term gives the Riemann curvature and the second gives the field terms that do not depend only on the metric. The conservation laws arise from varying the metric  $\delta g^{\mu\nu}$  from which we obtain

$$G^{\mu\nu} = 8\pi GT^{\mu\nu} = -\kappa \frac{-2}{\sqrt{-g}^{-1}} \frac{\delta \mathcal{L}_{\text{fields}}(\sqrt{-g})^{-1}}{\delta g^{\mu\nu}} \quad (14)$$

Since  $\nabla_\mu G^{\mu\nu} = 0$  as an identity we have  $\nabla_\mu T^{\mu\nu} = 0$ . This is a local conservation law. To obtain a global one we need a spacetime with persistent Killing vectors corresponding to continuous

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<sup>2</sup>Note that this does not mean that the energy of the rest field is  $m$  ( $c = 1$ ). The energy is a function of the triple of fields  $(\psi, \phi, \gamma)$  as we see next.



symmetries. Since the action of gravity typically destroys these global conservation laws. However, if  $G \rightarrow 0$  and the initial data is chosen to be flat then these exist and persist so we have the usual global symmetric conservation laws. This justifies this as a general method of deriving conservation laws with symmetric stress-energy tensors for fields on flat space when all the fields present are tensorial. Of course, we expect any such conservation law to correspond to a symmetry. In this case, we can vary the coordinates locally and this leaves the quantity  $\mathcal{L}\sqrt{g_{\cdot\cdot}}$  invariant. Since all the derivatives are covariant, we can replace a passive coordinate change on an open set with an active transformation of the metric. Varying the metric is therefore equivalent to a general small variation in the local coordinates.

The (symmetric) stress tensor is usually defined by<sup>3</sup>

$$T_{\mu\nu} = -\frac{2}{(\sqrt{-g^{\cdot\cdot}})^{-1}} \frac{\delta(\mathcal{L}_{\text{fields}}(\sqrt{-g^{\cdot\cdot}})^{-1})}{\delta g^{\mu\nu}} = -2\frac{\delta(\mathcal{L}_{\text{fields}})}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_{\text{fields}} \quad (15)$$

$$= 2i(\phi_a\gamma_{(\mu:ab}\partial_{\nu)}\psi_b - (\partial_{(\mu}\phi_a)\gamma_{\nu):ab}\psi_b) + g_{\mu\nu}(i(g^{\alpha\beta}\phi_a\gamma_{\alpha:ab}\partial_{\beta}\psi_b - g^{\alpha\beta}(\partial_{\alpha}\phi_a)\gamma_{\beta:ab}\psi_b) - 2m\phi_a\psi_a) \quad (16)$$

$$= 2i(\phi_a\gamma_{(\mu:ab}\partial_{\nu)}\psi_b - [\partial_{(\mu}\phi_a]\gamma_{\nu):ab}\psi_b) \quad (17)$$

where we have varied with respect to  $g^{\mu\nu}$  and assumed  $\gamma_{\mu}$  is a field independent of it in anticipation of  $g^{\mu\nu}$  being a function of  $\lambda^{\mu}$ .

We can similarly examine the continuous symmetry given by the globally constant phase changes  $\psi \rightarrow e^{i\theta}\psi$  and  $\phi \rightarrow e^{-i\theta}\phi$  to get the conserved current

$$j^{\nu} = 2ig^{\mu\nu}\phi_a\gamma_{\mu:ab}\psi_b \quad (18)$$

so that  $\nabla_{\nu}j^{\nu} = 0$ . Here we see this current also depends on all three fields so that the vanishing of any one of them on a region necessitates the entirety of the physical reality vanish.

We will now consider the implications of packet motion given these two conservation laws. Firstly, when we say “packet” we are not referring to a packet of localized  $\psi$  or  $\phi$  as much as a localized region where the reality associated with these fields through  $T_{\mu\nu}$  and  $j^{\mu}$  are nonzero. Let us also consider a packet that is devoid of internal stress and rotation and where the pressure is minimal. For such a packet with sufficiently uniform interior we can average over the current to give  $\langle j^{\mu} \rangle \approx mv^{\mu}$  where  $m^2$  is the averaged  $g_{\mu\nu}j^{\mu}j^{\nu}$  density and, assuming the packet preserves its structure as it moves,  $v^i$  is the local coordinate velocity of the packet. We can then define  $v^0$  by the relation  $g_{\mu\nu}v^{\mu}v^{\nu} = -1$ . The conservation law tells us that  $\rho$  is conserved.  $v^{\mu}$  is well defined to the extent packet motion is so.

From  $\langle T^{\mu 0} \rangle$  we can define a velocity  $u$  that carries the energy in a localized packet so that  $\langle T^{\mu 0} \rangle \approx m'u^{(\mu}u^{0)}$ . Since a vanishing of the current on a region implies vanishing of stress-energy as well we have that  $v = u$  and that  $\langle T^{\mu 0} \rangle \approx m'(\mu v^0) = \alpha m^{(\mu}v^0)$ . Since there are no internal stresses  $\langle T^{\mu\nu} \rangle \approx \alpha m v^{\mu}v^{\nu}$ . By combining these expressions we derive that these “macroscopic” variables are determined in terms of the conserved quantities by

$$v^{\nu} = \frac{\langle T^{\mu\nu} \rangle}{\alpha \langle j^{\mu} \rangle} \quad (19)$$

$$m = \alpha^2 \frac{\langle j^{\mu} \rangle \langle j^{\nu} \rangle}{\langle T^{\mu\nu} \rangle} \quad (20)$$

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<sup>3</sup>Here we make the choice of taking the determinant with respect to the contravariant metric in anticipation of later work. This explains the power -1 this expression.

Where these are actually several equations (repeated indices are not summed) that are all equal by the conditions above.

Now consider the parcel averaged stress-energy conservation law. Applying  $\nabla_\mu j^\mu = 0$  we have

$$\langle \nabla_\mu T^{\mu\nu} \rangle = \langle \nabla_\mu (j^\mu v^\nu) \rangle \quad (21)$$

$$= \langle (\nabla_\mu j^\mu) v^\nu + j^\mu \nabla_\mu v^\nu \rangle \quad (22)$$

$$= m' \langle \nabla_\nu v \rangle = 0 \quad (23)$$

which indicates the gauge invariant aspects (i.e. the reality) of the parcel follows geodesic motion. This is not entirely surprising given that it is known that the conservation laws generally dictate that classical particles follow geodesics.[7]

In our theory of “lepto-electro-gravity” we have two covariant gauge fields and one contravariant one. In this sense we think of it as a “2+1” theory. If we were to include the weak and strong forces it would be a “4+1” theory. One contravariant field is always necessary to match the covariant derivatives that must arise in any differential equation. The electron field is described by a  $(\phi, \psi)$  pair of fields that embody its reality with a very large gauge group and the meaning of the reality they describe depends not only the metric but the covariant gravity field  $\gamma_\mu$ . We will see that these have properties that are distinct from the positive energy positrons so we will require another pair of fields for their description. Along the way we will introduce a lagrangian that exists as a purely polynomial expression and removes the need for complicated nonanalytic measures and rational inverse matrix functions.

## 4.2 Electro-gravity Lagrangian

Here we seek a lagrangian that encompasses electrons, positrons, electromagnetism and gravity. We seek to have equations that are polynomial rather than complicated rationals that arise from the operation of taking the inverse of the metric. For this reason we define the function  $g : \mathcal{V} \rightarrow \mathcal{T}$  where  $\mathcal{V}$  is the set of vector-plet objects  $\lambda_{ab}^\mu$  and  $\gamma_{\mu:ab}$  and  $\mathcal{T}$  is the set of corresponding contravariant or covariant 2-tensors  $g^{\mu\nu}$  and  $g_{\mu\nu}$  respectively. Specifically,  $g(A, B) = -8^{-1} \text{Tr}(AB + BA)$ . We will establish a lagrangian that gives Dirac particle motion in the flat space limit, electromagnetism and a form for GR that gives a simple parallel between the motion of the gravitational fields,  $\gamma_\nu$  and the electromagnetic ones  $A_\nu$  that allows gravity to obtain the nonlinear “geometric” features of GR.

Since we are interested predominantly in positive energy solutions we will need to introduce a separate action term  $\Lambda_p$  for positrons that have positive mass but a reversal of sign of the charge in the coupling. We can write the lagrangian for the covariant gravitational field  $\gamma$  by substitution into the Einstein-Hilbert lagrangian. Alternately, we can choose it to have a similar form of the action  $\Lambda'_g$  as the other vector potential  $\Lambda_A$  and the coupling terms  $\Lambda_{e\lambda A}$ ,  $\Lambda_{p\lambda A}$  will involve both the contravariant gravitational field  $\lambda$  and the vector potential. Finally, there will need to be some way for the covariant and contravariant gravitational fields to relate to one another. This will be accomplished by a Higgs-like interaction term  $\Lambda_c$ . The general action is then defined as

$$S = \int d^4x \mathcal{L} \sqrt{-g} = \int d^4x \Lambda = \int d^4x (\Lambda_g + \Lambda_\lambda + \Lambda_A + \Lambda_e + \Lambda_p + \Lambda_{e\lambda A} + \Lambda_{p\lambda A} + \Lambda_c) \quad (24)$$

where we will define  $\Lambda_\lambda$  shortly.

Since the measure is a nonanalytic function of the metric but this is not retained in the usual equations of motion. We will find that this is also true here. For reasons as above we use the  $\lambda$  fields in defining the measure.

The electron part of the action is given by the substitutions

$$\Lambda_e = \mathcal{L}_e(\sqrt{g^{\cdot\cdot}(\lambda)})^{-1} = (i(g^{\mu\nu}(\lambda)\phi_a\gamma_{\mu:ab}\nabla_\nu\psi_b - g^{\mu\nu}(\lambda)(\nabla_\mu\phi_a)\gamma_{\nu:ab}\psi_b) - 2m\phi_a\psi_a)(\sqrt{g^{\cdot\cdot}(\lambda)})^{-1} \quad (25)$$

where we have, harmlessly, replace the ordinary with covariant derivatives since the act on spinlet objects which are essentially scalars. Variation with the measure present allows their action of higher tensors to give the appropriate covariant connection terms. This is one indication of how the physics itself can generate the geometric aspects of gravity rather than imposing it by fiat in the formulation of the theory's foundations.

The positron portions of the lagrangian is of the same form as  $\Lambda_e$  but with a different pair of fields  $\tilde{\phi}, \tilde{\psi}$ . The distinction comes in the form of the interaction terms. The usual minimal coupling prescription gives

$$\Lambda_{e\lambda A} = -q\phi_a\lambda_{ab}^\mu A_\mu\psi_b \quad (26)$$

$$\Lambda_{p\lambda A} = +q\tilde{\phi}_a\lambda_{ab}^\mu A_\mu\tilde{\psi}_b \quad (27)$$

It is only the sign of the charge in the interaction terms that distinguishes positrons from electrons and it only appears in the couplings.

The gravitational part of the action can be defined by a simple extension of the Einstein-Hilbert action

$$\Lambda_g = \frac{1}{2\kappa}R(g_{\mu\nu}(\gamma), g^{\mu\nu}(\lambda))(\sqrt{g^{\cdot\cdot}(\lambda)})^{-1} \quad (28)$$

$R$  is defined in terms of  $g_{\mu\nu}(\gamma)$ ,  $g^{\mu\nu}(\lambda)$  and the connections implicit in the expression are defined by  $\Gamma_{\mu\nu}^\alpha = 2^{-1}g^{\alpha\sigma}(\lambda)(g_{\mu\nu,\sigma}(\gamma) + g_{\sigma\mu,\nu}(\gamma) - g_{\nu\sigma,\mu}(\gamma))$  and their derivatives. We expect that some induced constraints force  $g(\gamma)g(\lambda) = \delta$ . To have this done as a result of field interactions we exploit a ‘‘Higgs-ish’’ mechanism with the coupling term

$$\Lambda_c = M|g_{\mu\nu}(\gamma)g^{\nu\rho}(\lambda) - \delta_\mu^\rho|^2 \quad (29)$$

for a sufficiently large mass  $M$ . When the energies in the other terms are much smaller this drives the relation between  $\gamma$  and  $\lambda$  to hold so that the solutions become ‘‘geometric.’’ Specifically, while it is easy to enforce causality if all evolution fields obey some equation such as  $g^{\mu\nu}\partial_\mu\partial_\nu\phi + \dots$  where  $g^{\mu\nu}$  is a metric with signature  $+2$ , the geometric case indicates that slowly spreading packets in regions of slowly varying spacetime move along geodesics. When such a relation holds our lagrangian has a form that can be interpreted as coordinate invariant in that the derivatives act on the tensor fields with covariant derivatives with the connections induced by the metric  $g_{\mu\nu} = -4^{-1}\text{Tr}\gamma_{(\mu}\gamma_{\nu)}$ . In the next section we will see that we can also interpret the system to live on a flat background and derive global conservation laws.

The other gauge fields all come from lagrangians that have electromagnetic form  $F^{\mu\nu}F_{\mu\nu}$  where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . Specifically,

$$\Lambda_A = g^{\mu\alpha}(\lambda)g^{\nu\beta}(\lambda)(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\alpha A_\beta - \partial_\beta A_\alpha)(\sqrt{g^{\cdot\cdot}(\lambda)})^{-1} \quad (30)$$

It is not necessary to use covariant derivatives here since antisymmetry cancels them. For example, we model the action contribution from the ‘‘dual field’’  $\lambda$  as

$$\Lambda_\lambda = \epsilon g^{\mu\alpha}(\lambda)g^{\nu\beta}(\lambda)\text{Tr}(\partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu)(\partial_\alpha\lambda_\beta - \partial_\beta\lambda_\alpha)\left(\sqrt{-g(\lambda)}\right)^{-1} \quad (31)$$

where  $\lambda_\mu = g_{\mu\nu}(\gamma)\lambda^\nu$ . where we have chosen the constant  $\epsilon$  to be small so that the dynamics can be dominated by  $\gamma$  and the constraints.

For a function  $F(g(\gamma))$  the variation under  $\delta\gamma_\nu$  gives

$$\delta F = \frac{\delta F}{\delta g_{\mu\nu}} \delta\gamma_\nu \quad (32)$$

and similarly for  $\delta\lambda$ . Variation of  $\Lambda_g$  by  $\delta\lambda$  gives

$$\frac{1}{2\kappa} (R_{\mu\nu} - 2^{-1} R g_{\mu\nu}) \delta\lambda_\nu \quad (33)$$

or

$$G_{\mu\nu} \delta\lambda_\nu = \kappa T_{\mu\nu} \delta\lambda_\nu \quad (34)$$

where  $T_{\mu\nu}$  is the stress-energy tensor for all the actions terms other than  $\Lambda_g$ . We have implicitly assumed that we are in a low enough energy regime and the initial data includes no “waves” of  $\lambda$  so that the contributions of  $\Lambda_\lambda$  can be ignored. Since the  $\gamma$ ’s contain gauge freedom that is independent of coordinate changes so that we can choose any  $\gamma_\mu$  that give the same  $g_{\mu\nu}(\gamma)$  field, this requires

$$G_{\mu\nu} = \kappa T_{\mu\nu} \quad (35)$$

Since we are interested in some sort of unification of fields, we would like the  $\gamma$  fields to enter in a form similar to  $A$ . The Riemann tensor satisfies the identity

$$\nabla_\alpha \nabla_\beta \gamma_\mu - \nabla_\beta \nabla_\alpha \gamma_\mu = R^\mu_{\nu\alpha\beta} \gamma_\mu \quad (36)$$

This suggests that there could be a variational approach to the Einstein equations in terms of  $\lambda$  and  $\gamma$  similar to the electromagnetic lagrangian. We consider

$$\Lambda'_g = \frac{1}{2\kappa} g^{\mu\alpha}(\lambda) g^{\nu\beta}(\lambda) \text{Tr}[(\partial_\nu \gamma_{(\alpha} \partial_\mu \gamma_{\beta)}) - \partial_\mu \gamma_{(\alpha} \partial_\nu \gamma_{\beta)}] \sqrt{-g..} \quad (37)$$

$$= \mathcal{L}'_g(\lambda, \gamma) m(\gamma) \quad (38)$$

In the case of eqn. 28, which was a second order expression in  $\gamma$ , we varied  $\lambda$  to obtain the equations of motion for  $\gamma$ . The variation with respect to  $\gamma$  gave contributions to the equations of motion for  $\lambda$  which is dominated by the constraint like contribution from  $\Lambda_c$ . Here we have a first order lagrangian in  $\gamma$  so we vary  $\gamma$  itself. The resulting equations are

$$\frac{1}{2\kappa} (\delta\gamma_\alpha R^{\sigma\alpha} \gamma_\sigma + \gamma_\alpha R^{\sigma\alpha} \delta\gamma_\sigma - 2^{-1} g^{\sigma\alpha} \mathcal{L}'_g (\delta\gamma_\alpha \gamma_\sigma + \gamma_\alpha \delta\gamma_\sigma)) \quad (39)$$

which by a similar gauge argument as above gives

$$G'^{\mu\nu} = (R^{\sigma\alpha} - 2^{-1} g^{\sigma\alpha} \mathcal{L}'_g) \quad (40)$$

This is the part that will then equal  $T^{\mu\nu}$  as defined by the variation  $S - S_{g'}$  with respect to  $g_{\mu\nu}$ . We know that  $\nabla_\mu T^{\mu\nu} = 0$  by the identity  $\nabla_\mu G^{\mu\nu} = 0$  which follows from the Bianchi identities. However, the variation of  $S - S_{g'}$  does not depend on  $S_g$ . That term can be neglected or replaced with any other function. This ensures that  $\nabla_\mu T^{\mu\nu} = 0$  holds generally so that  $\nabla_\mu G'^{\mu\nu} = 0$  so  $\mathcal{L}'_g = R$  up to a total divergence.

## 5 Conservation Laws

We can argue the whole structure exists on a flat background though this is just a convenient artifice among many. It is however a very convenient one. The appearance of geometric evolution via the additional terms that make the derivatives seem “covariant” with respect to some induced geometry of these fields is an emergent byproduct of the kind of couplings present. Of course, we still need to know if our equations can be evolved for arbitrary times using this point of view. Some discussion of this, especially in the case of black hole formation is given in [8]. For now we assume that this is unlimited however, although other methods have attempted to justify working on a flat background [6] it is a delicate process to have this make sense as gravitational collapse ensues due to the trend of the equations to become ill conditioned here. One should not be overly comfortable with formalism in this case. A method to handle evolution on the large regions of nearly degenerate metric using conservation laws is proposed in [8]

The flat background has a natural set of Killing vectors that give global conservation laws. To elucidate this consider the lagrangian written in terms of ordinary derivatives and make the modification

$$\Lambda = \mathcal{L}\sqrt{-g} \rightarrow \Lambda\sqrt{-\eta} \quad (41)$$

and all actions on tensors transform as

$$\partial_\mu A^\alpha \rightarrow \nabla_\mu(\eta)A^\alpha = \partial_\mu A^\alpha - \Gamma_{\mu\nu}^\alpha(\eta)A^\nu \quad (42)$$

and so forth, where  $\eta$  is a metric (in any coordinates) that can be varied about the flat space case. Any covariant derivatives  $\nabla_\mu(g)$  in terms of the metric induced connections are reinterpreted as formal couplings through  $\Gamma(g)$  and the  $\partial_\mu$  are converted by this prescription. We then modify introduce an extra faux-gravitational term built on the metric  $\eta$

$$\Lambda_\eta = \frac{1}{2\kappa'} R(\eta)\sqrt{-\eta} \quad (43)$$

and let  $\kappa' \rightarrow \infty$  after we vary to get the new stress tensor  $T'_{\mu\nu}$  where this now includes the gravitational fields  $\gamma$  and the associated  $\lambda$ . Since the flat space contains a full set of ten Killing vectors we have a set of conserved global quantities that now includes the gravitational fields of the form

$$\partial_\mu T'^{\mu\nu} = 0 \quad (44)$$

with the Killing (co)vector fields  $p_\nu = \hat{\omega}_\nu$ ,  $M_{ij} = x_i\hat{\omega}_j - x_j\hat{\omega}_i$  and  $b_i = x_0\hat{\omega}_i + x_i\hat{\omega}_0$ . The globally conserved quantities in these coordinates are

$$C = \int d^3x p_\mu T'^{\mu\nu} \quad (45)$$

$$J = \int d^3x \epsilon_{ijk} M_{ij} T'^{ik} \quad (46)$$

$$P = \int d^3x b_i T'^{i0} \quad (47)$$

## 6 Conclusion

The usual approaches to the Dirac equation and, by extension, particle physics in general, is through group theory and representations. This has been a powerful tool to limit the possible forms of the lagrangians. In relativity, the manifest coordinate invariance of expressions and local Lorentzian nature of the spacetime is codified in the metric evolution equations where the transformation properties of the fields and their derivatives are covariant and explicit functions of the metric. Here we have taken a different point of view in an attempt to form a union of the two. The fields couple in a simple fashion and can give a geometrodynamical picture in terms of some of the fields in the low energy limit even though we define there to be an underlying flat geometry. An interesting possibility is that since our lagrangian can be written entirely in terms of ordinary derivatives, it automatically generates covariant ones when the action is varied. Covariant derivatives have a different form for each tensor type they are applied to. The very form of the transformation laws for nontrivial tensors can then be induced by the form of the lagrangian itself.

One of the more interesting aspects of this theory is that the equations of motion in the flat space case for a Dirac particle are unchanged but our interpretation of them and the allowed gauge group is very different. It is sometimes objected that varying with  $\psi$  and  $\bar{\psi}$  in the usual Dirac Lagrangian seems redundant. It is not a priori consistent. There are two real fields that make up the complex field  $\psi_a$  (for fixed  $a$ ) but four if we treat the function and its dual as independent. Our introduction of a bilinear lagrangian removes this problem. The introduction of the Gupta-Bluer formalism and other operator formalisms have been introduced to handle the problem of negative energy solutions. Our lagrangian removes this by the introduction of a separate set of fields. Should this theory turn out to gain wide acceptance one would have to wonder if the emphasis on representation theory and such mathematical machinations have been misplaced.

Causality is rigorously true for this theory even in the limits where the theory ceases to be a metric one. Representation theory never arises as the increased gauge freedom allows Lorentz invariance among the conserved, hence observable, quantities automatically. This differs from geometric algebra approaches [5][6] where the algebra and invariance properties of the objects start with a mathematical meaning rather than being induced by the physics. The meaning of the mass scale  $M$  in the  $\Lambda_c$  part of the action indicates an energy scale for gravity to distinguish itself as a “geometric” force that seems to control the local geometry for the other fields.

There are undoubtedly many inequivalent such theories with the same low energy limit so we have presented only one of probably many such solutions. From here it is unclear how to extend this classical theory to a quantum one. The couplings are such that they determine the local notion of causality and it is not clear when or how well a perturbative scheme, which is generally built on free fields solutions, will work in the many body case. This is a direction for future work.

Even from the purely “classical” (i.e. one body field) point of view one can ask some interesting GR questions. The no-hair conjecture implies that no black hole can have a magnetic field. One has to wonder if one were to supply a continuous stream of matter that is all spin up electrons into a black hole or simply constructed one from such matter how the fields would disappear. These particles have (apparently singular) magnetic fields and it is not clear how this would be changed by falling into the black hole. One might hope that one could consider the solution to be a kind of “square-root” of the Kerr metric since the black hole would inherit a nonzero angular momentum from the electrons. The problem is that no known extension of the Kerr metric includes a magnetic field. In the strong field case, the strong interactions that relate  $\gamma$  and  $\lambda$  and give a metric theory can be overwhelmed. This may shed some light on the behavior near the event horizon, especially

for very small holes where the curvature is extreme.

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